DO NOW

pg 211; 66, 69, 70

4.2 Rolle's Theorem & The Mean Value Theorem

The Extreme Value Theorem states: that a continuous function on a closed interval must have both a maximum and a minimum.

* Can be at the endpoints

Rolle's Theorem gives: conditions to guarantee the existence of an extreme value on the interior of a closed interval.

Rolle's Theorem:

Let f be continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b).

If $f(a)=f(b)$, then there is at least one number c in (a,b) such that $f'(c)=0$.

(a) f is continuous on $[a, b]$ and differentiable on (a, b).

(b) f is continuous on $[a, b]$.

not differentiable at c

Example: Determine whether Rolle's Theorem can be applied to f on the closed interval $[a, b]$. If Rolle's Theorem can be applied, find all values of c in the open interval (a, b) such that $f'(c) = 0$.

1. $f(x) = x^2 - 5x + 4,\ [1, 4]$

This is continuous.
This is differentiable.

$f(1) = 1-5+4 = 0$

$f(4) = 16-20+4 = 0$

$f(1) = f(4)$ ✓

∴ Rolle's Theorem can be applied.

$f'(x) = 2x-5$

$0 = 2x-5$

$\frac{5}{2} = x$

$\boxed{c = \frac{5}{2}}$

2. $f(x) = \frac{x^2 - 1}{x},\ [-1, 1]$

This function is not continuous at $x=0$.

∴ Rolle's Theorem does not apply

* If I change the interval to $[1,3]$

This would be continuous.
This would be differentiable

$f(1) = 0$

$f(3) = \frac{8}{3}$

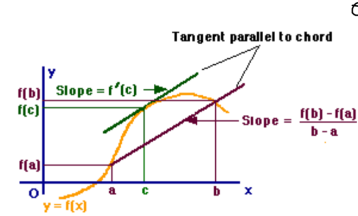
$f(1) \neq f(3)$

∴ Rolle's Theorem does not apply.

The Mean Value Theorem:

If f is continuous on the closed interval $[a, b]$ and differentiable on the open interval (a, b), then there exists a number c in (a, b) such that:

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$ ← slope of the secant line connecting the endpoints



There is a point where:

the instantaneous rate of change = the average rate of change

Example: Determine whether the Mean Value Theorem can be applied to f on the closed interval $[a, b]$. Find the values of c that satisfy the Mean Value Theorem.

3. $f(x) = \sqrt{x - 2}$, $[2, 6]$

This is continuous.
This is differentiable.
∴ The mean value theorem can be applied.

$$\frac{f(b)-f(a)}{b-a}$$

$$\frac{f(6)-f(2)}{6-2}$$

$$\frac{2-0}{4}$$

$$\frac{1}{2}$$

$$f'(c) = \frac{1}{2}$$

$$f(x) = (x-2)^{1/2}$$

$$f'(x) = \frac{1}{2}(x-2)^{-1/2}(1)$$

$$f'(x) = \frac{1}{2\sqrt{x-2}}$$

$$\frac{1}{2\sqrt{x-2}} = \frac{1}{2}$$

$$2\sqrt{x-2} = 2$$

$$\sqrt{x-2} = 1$$

$$x = 3$$

$$\boxed{c = 3}$$

4. $f(x) = \frac{x+1}{x}$, $[½, 2]$

This is continuous on this interval.
This is differentiable.
∴ The Mean Value Theorem can be applied.

$$\frac{f(b)-f(a)}{b-a}$$

$$\frac{f(2)-f(\frac{1}{2})}{2-\frac{1}{2}}$$

$$\frac{\frac{3}{2} - \frac{3/2}{1/2}}{\frac{3}{2}}$$

$$\frac{\frac{3}{2} - 3}{\frac{3}{2}}$$

$$\frac{-\frac{3}{2}}{\frac{3}{2}} = -1 = f'(x)$$

$$f'(x) = \frac{x(1) - (x+1)(1)}{x^2}$$

$$f'(x) = \frac{x - x - 1}{x^2}$$

$$f'(x) = \frac{-1}{x^2}$$

$$-1 = \frac{-1}{x^2}$$

$$-x^2 = -1$$

$$x^2 = 1$$

$$x = \pm 1$$

* -1 is not in the interval

$$\boxed{c = 1}$$

5. Two stationary patrol cars equipped with radar are 5 miles apart on a highway. As a truck passes the first patrol car, its speed is clocked at 55 mph. Four minutes later, when the truck passes the second patrol car, its speed is clocked at 50 mph. Prove that the truck must have exceeded the speed limit (of 55 miles per hour) at some time during the 4 minutes.

$$\frac{4 \text{ min}}{60 \text{ min}} = \frac{1}{15} \text{ hr}$$

1 — 4 min ($\frac{1}{15}$ hr) — 2
5 miles
55mph 50mph

$d(t)$ = distance in terms of time

$$d(0) = 0$$

$$d(\tfrac{1}{15}) = 5$$

$$\frac{d(\frac{1}{15}) - d(0)}{\frac{1}{15} - 0}$$

$$\frac{5-0}{\frac{1}{15}}$$

$$5 \cdot 15 = 75 \text{ mph}$$

* By the mean value theorem, at some point between the cops, the truck must have been traveling at 75mph.

HOMEWORK

pg 216 - 217; 11, 13, 16, 17, 21,
41 - 44, 50,